

Question 1

I conducted a market survey on the price of running shoe, and the detailed information is listed in the table below.

No	Brand	Model / Description	Price (S\$)	Source / URL
1	Nike	Air Jordan 1 Mid "Peach Mocha" DH0210-100	\$129	https://www.nikestores.us.org/air-jordan-1-mid-peach-mocha-dh0210100-p-14885.html
2	Nike	Off-White x Air Jordan 4 "Sail" CV9388-100	\$160	https://www.nikestores.us.org/offwhite-x-air-jordan-4-sail-cv9388100-p-15212.html
3	Nike	Nike Dunk High SE All-Star (2021) DD1398-300	\$149	https://www.nikestores.us.org/nike-dunk-high-se-allstar-2021-dd1398300-p-1243.html
4	Nike	NK x Sacai Blazer Low "Dark Grey" DD1877-002	\$138	https://www.nikestores.us.org/nk-x-sacai-blazer-low-dark-grey-dd1877002-p-1215.html
5	Nike	Nike Air Zoom GT Cut "Black" CZ0176-002	\$119	https://www.nikestores.us.org/nike-air-zoom-gt-cut-black-cz0176002-p-15433.html
6	Nike	Nike Air Yeezy 2 Pure Platinum 508214-010	\$270	https://www.nikestores.us.org/nike-air-yeezy-2-pure-platinum-508214010-p-1212.html
7	Nike	Off-White x Nike Air Rubber Dunk "University Blue" CU6015-100	\$140	https://www.nikestores.us.org/offwhite-x-nike-air-rubber-dunk-university-blue-cu6015100-p-15398.html
8	Nike	Off-White x Nike Air Presto "Black" AA3830002-	\$149	https://www.nikestores.us.org/offwhite-x-nike-air-presto-black-aa3830002-

		AA3830-002		p-15394.html
9	Nike	Nike Air More Uptempo "UK" AV3809-700	\$127	https://www.nikestores.us.org/nike-air-more-uptempo-uk-av3809700-p-15388.html
10	Nike	Nike Air Foamposite One "Floral" 314996-012	\$198	https://www.nikestores.us.org/nike-air-foamposite-one-floral-314996012-p-15217.html
11	Nike	Nike Air Max 1 Travis Scott Cactus Jack Wheat Lemon Drop DO9392-701	\$150	https://www.nikestores.us.org/nike-air-max-1-travis-scott-cactus-jack-wheat-lemon-drop-do9392701-p-1257.html
12	Nike	Nike Dunk SB Low Blue Lobster 313170-342	\$139	https://www.nikestores.us.org/nike-du nk-sb-low-blue-lobster-313170342-p-1230.html
13	Nike	Nike SB Dunk High Supreme By Any Means Navy DN3741-600	\$160	https://www.nikestores.us.org/nike-sb-dunk-high-supreme-by-any-means-navy-dn3741600-p-1228.html
14	Nike	Kaws x Sacai x Nike Blazer Low "Purple Dusk" DM7901-500	\$128	https://www.nikestores.us.org/kaws-x-sacai-x-nike-blazer-low-purple-dusk-dm7901500-p-15446.html
15	Nike	Nike Blazer Mid '77 "Celestine Blue" BQ6806-109	\$129	https://www.nikestores.us.org/nike-blazer-mid-77-celestine-blue-bq6806109-p-15454.html
16	Adidas	Ultraboost Light Shoes	\$190	https://www.adidas.com/us/ultraboost-light-shoes/GZ5159.html
17	Adidas	Adizero SL Running Shoes	\$96	https://www.adidas.com/us/adizero-sl-running-shoes/GY2589.html
18	Adidas	Duramo 10 Running	\$56	https://www.adidas.com/us/duramo-1

		Shoes		0-running-shoes/HP2380.html
19	Adidas	Solarglide 5 Running Shoes	\$78	https://www.adidas.com/us/solarglide-5-running-shoes/GX6703.html
20	Adidas	Adidas 4Dfwd Running Shoes	\$160	https://www.adidas.com/us/adidas-4d-fwd-running-shoes/IG5126.html
21	Adidas	Supernova 2.0 Running Shoes	\$60	https://www.adidas.com/us/supernova-2.0-running-shoes/GW9089.html
22	Adidas	Ultra 4Dfwd Running Shoes	\$176	https://www.adidas.com/us/ultra-4d-fwd-running-shoes/HP7599.html
23	Adidas	Adistar Cs Running Shoes	\$96	https://www.adidas.com/us/adistar-cs-running-shoes/GV9538.html
24	Adidas	Pureboost 22 Running Shoes	\$112	https://www.adidas.com/us/pureboost-22-running-shoes/HQ1450.html
25	Adidas	Runfalcon 2.0 Running Shoes	\$48	https://www.adidas.com/us/runfalcon-2.0-running-shoes/FZ2808.html
26	Adidas	4Dfwd Pulse 2 Running Shoes	\$96	https://www.adidas.com/us/4d-fwd-pulse-2-running-shoes/GX9281.html
27	Adidas	Adistar Running Shoes	\$78	https://www.adidas.com/us/adistar-running-shoes/GY1685.html
28	Adidas	Terrex Two Flow Trail Running Shoes	\$77	https://www.adidas.com/us/terrex-two-flow-trail-running-shoes/GY6145.html
29	Adidas	Adistar 2.0 Running Shoes	\$104	https://www.adidas.com/us/adistar-2.0-running-shoes/HP2335.html
30	Adidas	Adizero Prime X Strung	\$300	https://www.adidas.com/us/adizero-prime-x-strung/HQ3781.html

a) The mean = \$133.7333

Because the sample size $n = 30$, the median is the average value of the number in $n/2$ position and the number in $(n+2)/2$ position, after sequencing

the sample. Because $n = 30$, and the 15th and 16th numbers are both \$129, the median = \$129.

The mode = \$160

b) The standard deviation

$$\sigma = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (X_i - \bar{X})^2} = \sqrt{\frac{1}{30-1} \sum_{i=1}^{30} (X_i - 133.7333)^2} = 56.4495$$

c) The coefficient of variation = $\frac{\sigma}{\bar{X}} = \frac{56.4495}{133.7333} = 0.4221$

d) The first quartile is in the position of $(n+1)25/100 = 7.75$. We use the method of linear interpolation. Because the 7th number and the 8th number are both \$96, the first quartile = $0.25 \cdot 96 + 0.75 \cdot 96 = \96 . The second quartile is in the position of $(n+1)50/100 = 15.5$. Because the 15th number and the 16th number are both \$129, the second quartile = $0.5 \cdot 129 + 0.5 \cdot 129 = \129 . The third quartile is in the position of $(n+1)75/100 = 23.25$. Because the 23th number and the 24th number are both \$160, the third quartile = $0.25 \cdot 160 + 0.75 \cdot 160 = \160 . The interquartile range = the third quartile - the first quartile = $\$160 - \$96 = \$64$

Question 2

a)

i. The probability that a randomly selected customer who likes action movie = $110/200 = 0.55$

ii. The probability that a randomly selected customer who is male and like action movie = $80/200 = 0.4$

iii. The probability that a randomly selected customer who is female or like action movie = $(80+30+70)/200 = 0.9$

b)

i. Denote the salary of a sales manager as X , and $X \sim N(3500, 420^2)$. The probability that a randomly selected manager has a salary that is between \$3000 and \$4200 is $P(3000 \leq X \leq 4200) =$

$$P[(3000-3500)/420 \leq (X-3500)/420 \leq (4200-3500)/420]$$

$$= P(-1.19047619 \leq Z \leq 1.66667) = 0.952209648 - 0.116929641 = 0.83528.$$

$$Z \sim N(0,1)$$

ii. According to the central limit theorem, the average salary $\bar{X} \sim N(\mu, \sigma^2/n)$.

Because $n=20$, the probability that the average salary exceeds 3750 is $P(\bar{X} >$

$$3750) = P\left(\frac{\bar{X} - 3500}{\sqrt{\frac{420^2}{20}}} > \frac{3750 - 3500}{\sqrt{\frac{420^2}{20}}}\right)$$

$$= P(Z > 2.661985687) = 1 - 0.996115941 = 0.003884$$

Question 3

a) The sample mean calculated in Question 1 is 133.7333, and the sample standard deviation is 56.4495. Because the price of the product is normally distributed, we can apply the t distribution with the degrees of freedom of $n-1$ to calculate the confidence interval for the mean price. The confidence interval is

$$\bar{X} \pm t_{\frac{\alpha}{2}} \frac{s}{\sqrt{n}}. \text{ For 90\% confidence interval, and degrees of freedom} = 29,$$

$t_{0.05} = 1.699127$. The 90% confidence interval for the mean price is

$$133.7333 \pm 1.699127 \frac{56.4495}{\sqrt{30}} = (116.22175, 151.24492)$$

b) With reference to this context, the meaning of the confidence interval is that there is 90% probability that the population mean price of running shoe has a value in the range of (116.22175, 151.24492).

c) The sample mean = 133.7333, and $1.1 * 133.7333 = 147.1067$ determine whether the population mean price is above 147.1067 at the 0.05 level of significance.

The null hypothesis $H_0 : \mu \leq 147.1067$, the alternative hypothesis

$$H_a : \mu > 147.1067$$

The test statistics is $t = \frac{\bar{X} - 147.1067}{s/\sqrt{n}}$, with degrees of freedom 29.

The test statistics is $t = \frac{133.7333 - 147.1067}{56.4495/\sqrt{30}} = -1.2976$, and is smaller than

$t_{0.05} = 1.699127$, so we cannot reject the null hypothesis at the 0.05 level of significance. The population mean price is not 10% above the value of the sample mean at the 0.05 level of significance.

Question 4

a) A suitable case for simple linear regression analysis is to explore the relationship between the advertising budget and the sales units of a sunglasses company. The sales units and advertising expense for each month in the past year of this sunglasses company are listed in the table below.

Month	Sales Units	Advertising Expense(thousands,\$)
1	75	2
2	90	5
3	148	6
4	183	7
5	242	22
6	263	25
7	278	28
8	318	30
9	256	22
10	200	18
11	140	10
12	80	2

The dependent variable Y is the sales units, and the independent variable X is the advertising expense.

b) The regression equation is $Y_i = a + bX_i + \varepsilon_i, i = 1, \dots, 12$

Y_i is the sales units of month i , and X_i is the advertising expense of month i , a is the intercept term and b is the slope coefficient and ε_i is the residual for the month i observation.

The estimated form of the regression equation is $\hat{Y}_i = \hat{a} + \hat{b}X_i, i = 1, \dots, 12$

\hat{Y}_i is the estimated sales units given X_i , \hat{a} is the estimated intercept term, \hat{b} is the estimated slope coefficient.

$$\text{Because } \hat{b} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sum_{i=1}^n (X_i - \bar{X})^2} = \frac{9257.25}{1208.25} = 7.6617$$

\bar{Y} is the mean value of Y , \bar{X} is the mean value of X .

$$\hat{a} = \bar{Y} - \hat{b}\bar{X} = 189.4167 - 7.6617 \cdot 14.75 = 76.4066$$

The estimated form of the regression equation is

$$\hat{Y}_i = 76.4066 + 7.6617X_i, i = 1, \dots, 12$$

c) The correlation coefficient =

$$\frac{\text{Cov}(X, Y)}{\sqrt{\text{Var}(X)\text{Var}(Y)}} = \frac{\sum_{i=1}^n (X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum_{i=1}^n (X_i - \bar{X})^2} \sqrt{\sum_{i=1}^n (Y_i - \bar{Y})^2}} = \frac{\sum_{i=1}^n (X_i - 14.75)(Y_i - 189.4167)}{\sqrt{\sum_{i=1}^n (X_i - 14.75)^2} \sqrt{\sum_{i=1}^n (Y_i - 189.4167)^2}}$$

$$= 0.964452973$$

The correlation coefficient measures the linear relationship between X and Y , the closer the correlation coefficient is to 1, the stronger the linear relationship between X and Y is. The correlation coefficient in this case is 0.9645, and it means that there is a strong positive linear relationship between the sales units and the advertising expense of the sunglasses company.

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